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Constant Stress Accelerated Life Testing Using Rayleigh Geometric Process Model

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Abstract

The current study introduces the Rayleigh geometric process model for the analysis of accelerated life testing under constant stress. The geometric process describes a simple monotone process and has been applied to a variety of situations such as the maintenance problems in engineering. By assuming that the lifetime under increasing stress levels forms a geometric process, we derive the maximum likelihood estimators of the Rayleigh parameter in case of complete and censored data. For each type, we also derive confidence intervals for the parameters using asymptotic distribution. The performance of the estimators is evaluated by a simulation study with different pre-fixed parameters.'

Keywords: Geometric process, Maximum Likelihood Estimator, Fisher Information Matrix, Asymptotic Confidence Interval, Simulation Study.

Introduction

Development of highly sophisticated products, intense global competition, and increasing customer expectations has put new pressures on manufacturers to produce high-quality products. In order to ascertain the service life and performance of a product, life test under normal operating conditions is clearly the most reliable. The standard life testing methods would require a long period of time to obtain enough failure data necessary to make inferences. Hence, they are not suitable in above situations. For some products, it is possible to accelerate failures, and hence obtain failure information quickly by using the products more intensively than in usual case. According to such properties, the design and analysis of the Accelerated Life Test (ALT) are very important from a practical viewpoint.

ALT, generally deals with three types of stress loadings: constant stress, step stress and Progressive stress. Constant stress is the most common type of stress loading, in which every item is tested under a constant level of the stress, which is higher than normal level. In this kind of testing, we may have several stress levels, which are applied for different groups of the tested items. This means that every item is subjected to only one stress level until the item fails or the test is stopped for other reasons. If the stress level of the test is not high enough, many of the tested items will not fail during the available time and one has to be prepared to handle a lot of censored data. To avoid this problem, step-stress testing can be applied, in which, all items are first subjected to a specified constant stress for a specified period of time. Items that do not fail will be subjected to a higher level of stress for another specified time. The level of stress is increased step by step until all items have failed or the test stops for other reasons. Progressive-stress loading is quite like the step stress testing with the difference that the stress level increases continuously.

Failure data obtained from ALT can be divided into two categories: complete (all failure data are available) or censored (some of failure data are missing). Complete data consist of the exact failure time of test units, which means that the failure time of each sample unit is observed or known. In many cases when life data are analyzed, all units in the sample may not fail. This type of data is called censored or incomplete data. See for more details, Bagdonavicius and Nikulin [5], Meeker and Escobar [17], Nelson [19, 18], Mann and Singpurwalla [16].

with different type of data and planning has been studied by many authors. Pan et al. [20] proposed a bivariate constant stress accelerated degradation test model by assuming the parameter, a function of the stress levels. Yang [24] proposed an optimal design of 4-level constant-stress ALT. Fan and Yu [8] discuss the reliability analysis of the constant stress accelerated life tests in case of generalized gamma lifetime distribution. Chen et al. [6] discuss the optimal design of multiple CSALT plan on non-rectangle test region. Saxena et al. [22] discussed the case of Rayleigh lifetime distribution for step stress accelerated life testing (SSALT). Watkins and John [23] consider constant stress accelerated life tests based on Weibull distributions with constant shape and a log-linear link between scale and the stress factor. Ding et al. [7] dealt with Weibull distribution to obtain ALT sampling plans under type I progressive interval censoring with random removals. Ahmad et al. [3], Islam and Ahmad [10], Ahmad and Islam [2], Ahmad, et al. [4] and Ahmad [1] discuss the optimal constant stress accelerated life test designs under periodic inspection and Type-I censoring.

Definition

A stochastic process $\{X_n, n=1,2,...\}$ *is a geometric process (GP), if there exists a real* $\lambda > 0$ *such that* $\{\lambda^{n-1}X_n, n=1,2,...\}$ *forms a renewal process (RP). The number* λ *is called the ratio of the GP.*

The concept of GP is introduced by Lam [13] in the study of repair replacement problem. Large amount of studies in maintenance problems and system reliability have been shown that a geometric process model (GPM) is a good and simple model for analysis of data with a single trend or multiple trends, for example, Lam and Zhang [15], Lam [14] and Zhang [25]. So far, there are only three studies that utilize the GP in the analysis of ALT. Huang [9] introduced the GPM for the analysis of ALT with complete and censored exponential samples under the constant stress. Kamal et al. [12] extended the GPM for the analysis of ALT with complete Weibull failure data under constant stress. Zhou et al. [26] considered the GP implementation of the CSALT model based on the progressive Type-I hybrid censored Rayleigh failure data. Kamal et al. [11] used the GP for the analysis of CSALT for Pareto Distribution with complete data. Saxena et al. [21] studied the case of log-logistic GPM in case of censored data.

In this paper, the analysis of CSALT for Rayleigh distribution with complete and censored data by using the GPM is considered. Estimation of parameters is carried out by maximum likelihood (ML) technique. Asymptotic confidence intervals for parameters are also obtained. Statistical properties of estimates and confidence intervals are examined through a simulation study.

The Model

The Geometric Process

Let us define the GP. Suppose that $X_1, X_2, ..., X_n$ is a sequence of random variables. If there exists $\lambda > 0$ such that $\{\lambda^{n-1}X_n, n=1,2,...\}$ forms a renewal process (RP) with a constant mean μ, then $X_1, X_2, ..., X_n$ is called a GP and the real number λ is called the ratio of the GP.

It can easily be noted that a GP is stochastically increasing for $0 < \lambda < 1$ and stochastically decreasing in case of $\lambda > 1$. GPM can identify trend effects by two parameters: the mean μ of the underlying RP and the ratio λ which measures the direction and strength of a trend. With the inherent geometric structure, forecast using the GPM is simple and straightforward.

Mean and Variance of a Geometric Process:

It can be shown that if $\{X_n, n=1,2,3,...\}$ is a GP and the pdf of X_1 is $f(x)$ with mean μ and variance σ^2 then the pdf of X_n will be

$$f(X_n) = \lambda^{n-1} f(\lambda^{n-1}x), \qquad n=1,2,3,...$$

with $E(X_n) = \dfrac{\mu}{\lambda^{n-1}}$

and $Var(X_n) = \dfrac{\sigma^2}{\lambda^{2(n-1)}}$

Thus λ, μ and σ^2 are three important parameters of a GP.

The Rayleigh Distribution

The Rayleigh distribution has played an important role in modeling the lifetime of random phenomena. It arises in many areas of applications, including reliability, life testing and survival analysis.

The life time of a product at any level of stress is assumed to follow Rayleigh distribution with scale parameter θ. The probability density function (p.d.f.) of Rayleigh distribution is given by

$$f(x) = \frac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}}, \qquad 0 \le x < \infty, \theta > 0 \qquad (1)$$

Cumulative distribution function (c.d.f) is given by

$$F(x) = 1 - e^{-\frac{x^2}{2\theta^2}}, \qquad 0 \le x < \infty, \theta > 0 \qquad (2)$$

The corresponding survival function is

$$S(x) = e^{-\frac{x^2}{2\theta^2}}, \qquad 0 \le x < \infty, \theta > 0 \qquad (3)$$

The hazard function of x, denoted as $h(x) = f(x)/S(x)$ is obtained as

$$h(x) = \frac{x}{\theta^2}, \qquad 0 \le x < \infty, \theta > 0$$

Assumptions

The geometric model for ALT is based on the following assumptions:

(1) Suppose that an ALT under $z_k, k=1,2,..., s$, arithmetically increasing stress levels is

performed. A random sample of $N_i, i = 1,2,..., n$, identical items is placed under each stress level and start to operate at the same time. Whenever an item fails, it is removed from the test and its observed failure time x_{ki} is recorded.

(2) At any constant stress level, the product lifetime has a single parameter Rayleigh distribution.

(3) Let the sequence of random variables $X_0, X_1, X_2,..., X_s$, denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress. We assume $\{X_k, k = 0,1,2,..., s\}$ is a GP with ratio $\lambda > 0$.

Based on the definition given in subsection 2.1, if density function of X_0 is $f(x)$, then the pdf of X_k will be given by

$$f(X_k) = \lambda^k f(\lambda^k x), \qquad k = 0,1,2,\cdots,s$$

Therefore the pdf of a product lifetime (following Rayleigh distribution) at the k^{th} stress level is

$$f_{x_k}(x \mid \theta, \lambda) = \frac{\lambda^{2k} x}{\theta^2} \exp\left\{-\frac{\lambda^{2k} x^2}{2\theta^2}\right\} \qquad (4)$$

Maximum Likelihood Estimation

Case (i) Complete Data

Here the ML method of estimation is used because ML method is very robust and gives the estimates of parameter with good statistical properties. In this method, the estimates of parameters are those values which maximize the sampling distribution of data. The ML estimation method is very appropriate for one parameter distributions and also its implementation in ALT is mathematically more intense. Generally, estimates of parameters do not exist in closed form, therefore, numerical techniques such as Newton Method and some computer programs are used to compute them.

The likelihood function for CSALT for the Rayleigh distribution using GPM can be written as

$$L(x \mid \theta, \lambda) = \prod_{k=1}^{s}\prod_{i=1}^{n} f(x \mid \lambda, \theta)$$

$$= \prod_{k=1}^{s}\prod_{i=1}^{n} \frac{\lambda^{2k} x_{ki}}{\theta^2} \exp\left\{-\frac{\lambda^{2k} x_{ki}^2}{2\theta^2}\right\} \qquad (5)$$

It is usually easier to maximize the logarithm of the likelihood function rather than the likelihood function itself. Therefore, by taking the logarithm of the likelihood function, (5) becomes

$$l = \log L(x \mid \theta, \lambda) = \sum_{k=1}^{s}\sum_{i=1}^{n} \left\{2k \log\lambda + \log x_{ki} - 2\log\theta - \frac{\lambda^{2k} x_{ki}^2}{2\theta^2}\right\} \qquad (6)$$

The maximum likelihood estimators of λ, and θ can be obtained by solving $\frac{\partial l}{\partial \lambda} = 0$, and $\frac{\partial l}{\partial \theta} = 0$ respectively for λ, and θ where the values of $\frac{\partial l}{\partial \lambda}$, and $\frac{\partial l}{\partial \theta}$ are given as

$$\frac{\partial l}{\partial \lambda} = \sum_{k=1}^{s}\sum_{i=1}^{n} \left\{\frac{2k}{\lambda} - \frac{k\lambda^{(2k-1)} x_{ki}^2}{\theta^2}\right\} \qquad (7)$$

$$\frac{\partial l}{\partial \theta} = \sum_{k=1}^{s}\sum_{i=1}^{n} \left\{-\frac{2}{\theta} + \frac{\lambda^{2k} x_{ki}^2}{\theta^3}\right\} \qquad (8)$$

From equations (7) and (8), it is observed that these equations are non linear. Therefore, the closed forms of MLEs of λ, and θ do not exist. So, Newton-Raphson method must be used to solve these equations simultaneously to obtain the MLEs of λ, and θ.

Case (ii) Censored Data

For Type I censoring scheme, the test at each stress level terminates at time t. An item's exact failure time is observed only if its lifetime is $x_{ki} \le t$. It is assumed that at the k^{th} stress level $r_k (\le n)$ failures are observed before the test is suspended. Correspondingly, $(n-r_k)$ units survive the entire test without failing. The observed ordered failure times under the k^{th} stress level can be written as $x_{k(1)} \le x_{k(2)} \le \cdots \le x_{k(r_k)}$. Here, t is fixed in advance and r_k is random and therefore the likelihood function using GP for the Rayleigh distribution under CSALT for Type I censored data is given by

$$L_k(x \mid \lambda, \theta) = \frac{n!}{(n-r_k)!}\left[\prod_{i=1}^{r_k} f_X(x_{(i)})\right]\left[S_{X_k}(t)\right]^{(n-r_k)} \qquad (9)$$

where, $S_{X_k}(t)$ is the probability that an item is censored at time t and

$$S_{X_k}(t) = \exp\left\{-\frac{(\lambda^k t)^2}{2\theta^2}\right\} \qquad (10)$$

Using eq. (10), the likelihood function for one of the stress levels corresponding to eq. (1) for obtaining the ML estimates of λ and θ is given by

$$L_i(x \mid \lambda, \theta) = \frac{n!}{(n-r_k)!}\left[\prod_{i=1}^{r_k} \frac{\lambda^{2k} x}{\theta^2}\exp\left\{-\frac{(\lambda^k x)^2}{2\theta^2}\right\}\right]\left[\exp\left\{-\frac{(\lambda^k t)^2}{2\theta^2}\right\}\right]^{(n-r_k)}, \qquad (11)$$

It follows that the likelihood function of observed data in a total s stress levels is:

$$L_k(x \mid \lambda, \theta) = L_1 \times L_2 \times \cdots \times L_s$$

$$= \prod_{k=1}^{s} \frac{n!}{(n-r_k)!}\left[\prod_{i=1}^{r_k} \frac{\lambda^{2k} x_{k(i)}}{\theta^2}\exp\left\{-\frac{(\lambda^k x_{k(i)})^2}{2\theta^2}\right\}\right]\left[\exp\left\{-\frac{(\lambda^k t)^2}{2\theta^2}\right\}\right]^{(n-r_k)}$$

$$0 \le x_{k(1)} \le x_{k(2)} \le \cdots \le x_{k(r_k)} \le t; 1 \le k \le s \qquad (12)$$

The log-likelihood function corresponding to (12) takes the form

$$l = \log L(x \mid \lambda, \theta)$$

$$= \sum_{k=1}^{s} \log \frac{n!}{(n-r_k)!} + \sum_{k=1}^{s}\sum_{i-1}^{r_k}\left\{2k\log\lambda + \log x_{k(i)} - 2\log\theta - \frac{(\lambda^k x_{k(i)})^2}{2\theta^2}\right\} - \sum_{k=1}^{s}\frac{(n-r_k)(\lambda^k t)^2}{2\theta^2} \qquad (13)$$

The first order derivatives of $\log L(x \mid \lambda, \theta)$ are given by

$$\frac{\partial l}{\partial \lambda} = \sum_{k=1}^{s}\sum_{i=1}^{r_k}\left\{\frac{2k}{\lambda} - \frac{k x_{k(i)}^2}{\theta^2}\lambda^{2k-1}\right\} - \sum_{i=1}^{r_k}\frac{2k(n-r_k)(\lambda^k t)^2}{\lambda} \qquad (14)$$

$$\frac{\partial l}{\partial \theta} = \sum_{k=1}^{s}\sum_{i=1}^{r_k}\left\{\frac{x_{k(i)}^2}{\theta^3}\lambda^{2k} - \frac{2}{\theta}\right\} + \sum_{k=1}^{s}\frac{(n-r_k)(\lambda^k t)^2}{\theta^3} \qquad (15)$$

The equations (14) and (15) are quite complex in form to be solved. So, the Newton-Raphson method is used to solve these equations simultaneously to obtain $\hat{\lambda}$ and $\hat{\theta}$.

Asymptotic Confidence Intervals

According to large sample theory, the ML estimators, under some appropriate regularity conditions, are consistent and normally distributed. Since ML estimates of parameters are not in closed form, therefore, it is impossible to obtain the exact confidence intervals, so asymptotic confidence intervals based on the asymptotic normal distribution of ML estimators instead of exact confidence intervals are obtained here.

The Fisher-information matrix composed of the negative second order partial derivatives of log likelihood function can be written as

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$$F = \begin{bmatrix} \hat{I}_{11} & \hat{I}_{12} \\ \hat{I}_{21} & \hat{I}_{22} \end{bmatrix}$$

Where in case (i)

$$\hat{I}_{11} = \left(-\frac{\partial^2 l}{\partial \lambda^2} \right) = \sum_{k=1}^{s} \sum_{i=1}^{n} \left\{ \frac{2k}{\lambda^2} + \frac{k(2k-1)\lambda^{(2k-2)} x_{ki}^2}{\theta^2} \right\}$$

$$\hat{I}_{22} = \left(-\frac{\partial^2 l}{\partial \theta^2} \right) = \sum_{k=1}^{s} \sum_{i=1}^{n} \left\{ \frac{3\lambda^{2k} x_{ki}^2}{\theta^4} - \frac{2}{\theta^2} \right\}$$

$$\hat{I}_{12} = \left(-\frac{\partial^2 l}{\partial \theta \partial \lambda} \right) = \hat{I}_{21} = \sum_{k=1}^{s} \sum_{i=1}^{n} \left\{ -\frac{2k\lambda^{(2k-1)} x_{ki}^2}{\theta^3} \right\}$$

and in case (ii)

$$\hat{I}_{11} = \left(-\frac{\partial^2 l}{\partial \lambda^2} \right) = \sum_{k=1}^{s} \sum_{i=1}^{r_k} \left\{ \frac{2k}{\lambda^2} + \frac{k(2k-1)x_{k(i)}^2}{\theta^2} \lambda^{2(k-1)} \right\} + \sum_{i=1}^{r_k} \frac{2k(2k-1)(n-r_k)(\lambda^k t)^2}{\lambda^2}$$

$$\hat{I}_{22} = \left(-\frac{\partial^2 l}{\partial \theta^2} \right) = \sum_{k=1}^{s} \sum_{i=1}^{r_k} \left\{ \frac{3x_{k(i)}^2}{\theta^4} \lambda^{2k} - \frac{2}{\theta^2} \right\} + \sum_{k=1}^{s} \frac{3(n-r_k)(\lambda^k t)^2}{\theta^4}$$

$$\hat{I}_{12} = \left(-\frac{\partial^2 l}{\partial \theta \partial \lambda} \right) = \hat{I}_{21} = \sum_{k=1}^{s} \sum_{i=1}^{r_k} \left\{ -\frac{2kx_{k(i)}^2}{\theta^3} \lambda^{2k-1} \right\} - \sum_{k=1}^{s} \frac{2k(n-r_k)(\lambda^k t)^2}{\theta^3 \lambda}$$

Now, the variance covariance matrix can be written as

$$\Sigma = \begin{bmatrix} \hat{I}_{11} & \hat{I}_{12} \\ \hat{I}_{21} & \hat{I}_{22} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\lambda}) & ACov(\hat{\lambda}\hat{\theta}) \\ ACov(\hat{\lambda}\hat{\theta}) & AVar(\hat{\theta}) \end{bmatrix}$$

The $100(1-\gamma)\%$ asymptotic confidence interval for and λ, and θ are then given respectively as

$$\left[\hat{\lambda} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\lambda})} \right], \text{ and } \left[\hat{\theta} \pm Z_{1-\frac{\gamma}{2}} \sqrt{AVar(\hat{\theta})} \right],$$

Simulation Study

To assess the performance of the methods described in present study, a number of data sets with sample sizes $n = 50, 100, ..., 250$ are generated from Rayleigh distribution. The values for parameters and stress levels are chosen to be $\lambda = 3.7502$, $\theta = 0.50$ and $s = 2$ *and* 3. For different given samples and stress levels, the ML estimates, Mean squared errors (MSEs), absolute relative biases (RBias), Relative Error (RE), and the 95% asymptotic confidence intervals for λ, and θ are obtained by using the present GP model using the Newton-Raphson iteration procedure. The results of the estimates for λ, and θ based on 1000 replications are summarized in Table 1 and 2 for case (i) and in Table 3 and 4 for case (ii) respectively.

Table 1: Simulation study results with $\lambda = 3.7502$, $\theta = 0.5000$, and $s = 2$ for complete data

Sample Size	Parameters	Estimates	MSE	RAB	RE	LCL	UCL
50	λ θ	3.7512 0.4823	0.0725 0.0407	0.0003 0.0354	0.0718 0.4036	3.2235 0.0883	4.2789 0.8763
100	λ θ	3.6573 0.5076	0.0822 0.0413	0.0248 0.0152	0.0765 0.4062	3.1256 0.1098	4.1890 0.9054
150	λ θ	3.7528 0.5098	0.0745 0.0440	0.0007 0.0196	0.0728 0.4195	3.2178 0.0991	4.2878 0.9205
200	λ θ	3.7534 0.5123	0.0753 0.0442	0.0009 0.0246	0.0732 0.4202	3.2156 0.1012	4.2912 0.9234
250	λ θ	3.7543 0.5058	0.0756 0.0445	0.0011 0.0116	0.0733 0.4220	3.2154 0.0923	4.2932 0.9193

Table 2: Simulation study results with $\lambda = 3.7502$, $\theta = 0.5000$, and $s = 3$ for complete data

Sample Size	Parameters	estimates	MSE	RAB	RE	LCL	UCL
50	λ θ	3.7661 0.4870	0.0155 0.0172	0.0042 0.0260	0.0331 0.2621	3.5245 0.2314	4.0077 0.7426
100	λ θ	3.7652 0.4928	0.0101 0.0698	0.0039 0.0144	0.0268 0.5282	3.5702 -0.0247	3.9602 1.0102
150	λ θ	3.7589 0.4936	0.0101 0.0752	0.0023 0.0128	0.0268 0.5486	3.5629 -0.0439	3.9549 1.0311
200	λ θ	3.7572 0.4958	0.0165 0.0634	0.0019 0.0084	0.0343 0.5037	3.5054 0.0023	4.0089 0.9893
250	λ θ	3.7568 0.4952	0.0092 0.0689	0.0018 0.0096	0.0256 0.5251	3.5688 -0.0182	3.9448 1.0086

Table 3: Simulation study results with $\lambda = 3.7502$, $\theta = 0.5000$, and $s = 2$ for censored data

Sample Size	Parameters	estimates	MSE	RAB	RE	LCL	UCL
50	λ θ	3.7652 0.5182	0.0610 0.0158	0.0040 0.0364	0.0659 0.2516	3.2819 0.2742	4.2484 0.7622
100	λ θ	3.7673 0.5038	0.0638 0.0148	0.0046 0.0076	0.0673 0.2434	3.2734 0.2654	4.2612 0.7422
150	λ θ	3.7598 0.4918	0.0765 0.0089	0.0026 0.0164	0.0737 0.1883	3.2184 0.3079	4.3016 0.6757
200	λ θ	3.7568 0.4843	0.0619 0.0164	0.0018 0.0314	0.0663 0.2565	3.2692 0.2348	4.2444 0.7338
250	λ θ	3.7587 0.4947	0.0648 0.0153	0.0023 0.0106	0.0679 0.2476	3.2602 0.2523	4.2572 0.7371

Table 4: Simulation study results with $\lambda = 3.7502$, $\theta = 0.5000$, and $s = 3$ for censored data

Sample Size	Parameters	estimates	MSE	RAB	RE	LCL	UCL
50	λ θ	3.7595 0.4914	0.0183 0.0040	0.0025 0.0172	0.0361 0.1261	3.4951 0.3689	4.0239 0.6138
100	λ θ	3.7542 0.4928	0.0179 0.0657	0.0011 0.0144	0.0357 0.5125	3.4919 -0.0092	4.0164 0.9948
150	λ θ	3.6823 0.4967	0.0178 0.0078	0.0181 0.0066	0.0356 0.1767	3.4571 0.3236	3.9075 0.6698

200	λ θ	3.6624 0.5069	0.0191 0.0092	0.0234 0.0138	0.0369 0.1923	3.4531 0.3189	3.8716 0.6948
250	λ θ	3.6989 0.5112	0.0128 0.0164	0.0137 0.0224	0.0302 0.2563	3.5009 -0.1991	3.8968 0.3014

Discussion and Conclusion

This paper deals with use of GP model in the analysis of constant stress ALT plan for Rayleigh distribution with complete data as well as censored data. The MLEs, MSEs, RBias, and RE of the model parameters were obtained. Based on the asymptotic normality, the 95% asymptotic confidence intervals of the model parameters were also obtained in both the cases.

It is observed that the estimates obtained in the simulation study are very close to the true values of the parameters and are also quite well with relatively small mean squared errors. In the whole study, the parameters are estimated for different cases and it is found that as the sample size increases, the MSE gets smaller. It implies that a larger sample size results in a better large sample approximation. Hence, it can be said that the proposed GPM can be used in the analysis of ALT.

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